P114 SOLUTION SET 9

29.18. a) The emf is given by -

$$E = -N\frac{d\phi}{dt} = -75\frac{d}{dt}\left(8.8t - 0.51t^3\right)\times 10^{-2}\ \mathrm{T\,m^2}$$

$$= (-6.6 + 1.1457t^2)\ \mathrm{V}$$

b) $E(t=1) = -6.6 + 1.1457\times(1)^2 \approx -5.5\,\mathrm{V}$

$E(t=4) = -6.6 + 1.1457\times 4^2 \simeq 12\,\mathrm{V}$

29.19. The energy given out is the power in the circuit times the amount of time it dissipates energy.

$\mathcal{E} = \text{emf} = -\frac{\Delta\phi}{\Delta t}$ $\quad P = \frac{\mathcal{E}^2}{R}$

$$E = P\Delta t$$

$$= \frac{\mathcal{E}^2}{R}\Delta t = \left(\frac{\Delta\phi}{\Delta t}\right)^2\frac{\Delta t}{R}$$

$$= \frac{(\Delta\phi)^2}{R\Delta t}$$

$\Delta\phi = \text{Area}\times\Delta B = \pi(0.125\mathrm{m})^2(0.4\mathrm{T})$

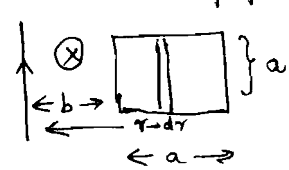
$$= \frac{\pi^2\times(0.125\mathrm{m})^4(0.4\mathrm{T})^2}{(150\,\Omega)(0.12\,\mathrm{s})} = 2.1\times 10^{-5}\ \mathrm{J}$$

29.22. The magnetic field inside a solenoid is given by $B = \mu_0 n I$.
The induced emf is given by $E = -\frac{d\Phi}{dt} = -A_1\frac{dB_{solenoid}}{dt}$

$$= -A_1\mu_0 n\frac{dI}{dt} = -A_0\mu_0 n\frac{d}{dt}(I_0\cos\omega t)$$

$$= A_0\mu_0 n\,\omega I_0\sin\omega t$$

29.25. a) The magnetic field produced by the wire is into the paper and given by -



$$\vec{B} = \frac{\mu_0}{4\pi}\times\frac{2I}{r}$$

Flux produced $= \int\vec{B}\cdot d\vec{A}$

$$= \int\frac{\mu_0}{2\pi}\frac{I}{r}\,a\,dr = \frac{\mu_0 I a}{2\pi}\int_b^{a+b}\frac{dr}{r}$$

$$= \frac{\mu_0 I a}{2\pi}\ln\left(\frac{b+a}{b}\right)$$

b) The speed of the loop is given by $V = \frac{db}{dt}$, the rate at which dist. between wire and loop changes.

$$E = -\frac{d\phi}{dt} = -\frac{\mu_0 I a}{2\pi}\frac{d}{dt}\left[\ln\left(1+\frac{a}{b}\right)\right] = -\frac{\mu_0 I a}{2\pi}\frac{1}{\left(1+\frac{a}{b}\right)}\times\left(-\frac{a}{b^2}\right)\times\frac{db}{dt}$$

$$= \frac{\mu_0 I a}{2\pi} \times \frac{a}{\frac{(b+a)}{b}} \times \frac{1}{b^2} \times V$$

$$= \frac{\mu_0 I a^2 V}{2\pi b(b+a)}$$

c) The produced magnetic field is into the paper and decreasing. The induced current will be clockwise to create a downward magnetic field.

d) Power dissipated is given by -

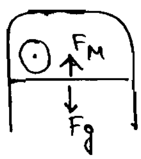
$$P = \frac{E^2}{R}$$

$$F = \frac{P}{V} = \frac{E^2}{RV} = \frac{\mu_0^2 I^2 a^4 V^2}{4\pi^2 b^2 (a+b)^2 V R} = \frac{\mu_0^2 I^2 a^4 V}{4\pi b^2 (a+b)^2 R}$$

29.28. $E = l\,\vec{B} \times \vec{V} = l B V \sin 90° = B l V$

$$V = \frac{E}{Bl} = \frac{0.12\,V}{(0.9\,T)(0.132\,m)} = 1\ m/sec$$

29.31.



The rod attains the terminal velocity when the magnetic and gravitational force are equal to each other.

$$F_M = \frac{B^2 l^2 V}{R} = mg$$

$$V = \frac{mgR}{B^2 l^2} = \frac{(3.6 \times 10^{-3}\,kg)(9.8\,m/s^2)(0.0013\,\Omega)}{(0.06\,T)^2\,(0.18\,m)^2} = 0.39\,m/s$$

29.47. we know that -

$$\frac{N_s}{N_p} = \frac{V_s}{V_p}$$ (N_s, N_p denote no. of coils in secondary, primary coil; V_s, V_p " voltage " " ")

$$= \frac{12\,000}{240} = 50$$

On ~~reversing~~ connecting backward, the primary and secondary coils are exchanged i.e.

$$\frac{N_s}{N_p} = \frac{1}{50} = \frac{V_s}{240} \qquad \therefore\ V_s = \frac{240\,V}{50} = 4.8\,V$$

29.67. $I = \frac{E}{R}$

$$E = -N_{coil}\frac{d\phi}{dt}$$

The magnetic field inside a solenoid $B = \mu_0 \frac{I_{sol}N}{l}$

$$\phi = B \times \text{Area}$$

$$\frac{d\phi}{dt} = A \times \frac{d}{dt}\left(\frac{\mu_0 I_{sol} N}{l}\right) = \frac{\mu_0 I A N}{l}\frac{dI_{sol.}}{dt}$$

$$I = N_{coil}\frac{\mu_0 I A N}{l R}\frac{dI_{sol.}}{dt}$$

$$= \frac{(150 \text{ turns})\pi (0.045 m)^2 (4\pi \times 10^{-7} \text{ T m/A})(230 \text{ turns})}{12\,\Omega \times 0.01 m} \times \frac{2A}{0.1s}$$

$$= 4.6 \times 10^{-2} A$$

On increasing current in solenoid, the magnetic increases from right to left. The induced current must flow from left to right across the resistor to create an opposing magnetic field.

29.68. The emf produced is the average emf over a whole cycle during which the coil changes its orientation and so also the flux.

$$E_{avg.} = -N\frac{\Delta\phi_B}{\Delta t} = -NA\frac{\Delta B}{\Delta t} = -NA\frac{((-B)-(+B))}{\Delta t}$$

due to change by rotation magnetic field changes from $-B$ to $+B$.

$$= \frac{2NAB}{\Delta t}$$

$$Q = I\Delta t = I \times \frac{2NAB}{E_{avg.}} = \frac{E_{avg.}}{R} \times \frac{2NAB}{E_{avg.}}$$

$$(\because I = \frac{E_{avg.}}{R})$$

$$\therefore Q = \frac{2NAB}{R} \Rightarrow B = \frac{QR}{2NA}$$

30.1. a) The mutual inductance is given by -

$$M = \frac{\mu N_1 N_2 A}{l} = \frac{1850 (4\pi \times 10^{-7} \text{ Tm/A})(225)(115)\pi (0.02m)^2}{2.44 m}$$

$$= 3.1 \times 10^{-2} H$$

b) The emf induced in the coil is -

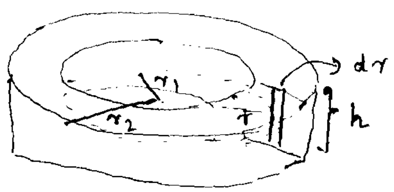
$$E = -M\frac{dI_1}{dt} = -3.1 \times 10^{-2} \times \frac{(-12A)}{0.098 ms} = 3.79 V$$

30.6. The inductance of a solenoid is given by -

$$L = \frac{\mu_0 N^2 A}{l}$$

$$N = \sqrt{\frac{Ll}{\mu_0 A}} = \sqrt{\frac{(0.13H)(0.3m)}{(4\pi \times 10^{-7}\,Tm/A)\,\pi\,(0.021m)^2}} \simeq 4700 \text{ turns}$$

30.13. $L = \frac{N}{I}\Phi_B$



The flux across the whole needs to be calculated using the magnetic field inside it as -

$$B(r) = \frac{\mu_0 N I}{2\pi r}$$

$$L = \frac{N}{I}\int_{r_1}^{r_2} \frac{\mu_0 N I}{2\pi r}\underbrace{h\,dr}_{\text{Area}} = \frac{\mu_0 N^2 h}{2\pi}\int_{r_1}^{r_2}\frac{dr}{r}$$

$$= \frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{r_2}{r_1}\right)$$

30.15. The magnetic field energy density is given by -

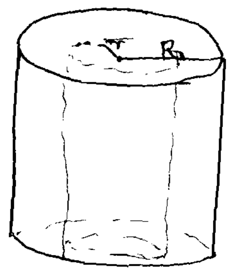
$$U = \frac{\text{Energy stored}}{\text{Volume}} = \frac{1}{2}\frac{B^2}{\mu_0}$$

$$\text{Energy stored} = \frac{1}{2}\frac{B^2}{\mu_0} \times \text{Volume} = \frac{1}{2}\frac{B^2}{\mu_0}\underbrace{\pi r^2 l}_{\text{Volume of cylinder}}$$

$$= \frac{1}{2}\frac{(0.6T)^2}{(4\pi \times 10^{-7}\,Tm/A)}\pi(0.0105m)^2(0.380m)$$

$$= 18.9\,J$$

30.21.



We find the magnetic field inside using Ampere's law -

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl.}$$

$$B \times 2\pi r = \mu_0 I \times \frac{\pi r^2}{\pi R^2}$$

$$B = \frac{\mu_0 I r}{2\pi R^2}$$

$$\text{Energy density per unit length} = \frac{U}{l} = \frac{1}{l}\int u_B\,dV = \int \frac{B^2}{2\mu_0} \times \overbrace{2\pi r\,dr}^{dV}$$

$$= \int \frac{\mu_0^2 I^2 r^2}{4\pi^2 R^4} \times \frac{1}{2\mu_0} 2\pi r\,dr = \frac{\mu_0 I^2}{4\pi R^4}\int_0^R r^3\,dr$$

$$= \frac{\mu_0 I^2}{4\pi R^4} \times \frac{R^4}{4} = \frac{\mu_0 I^2}{16\pi}$$